Section 5.5: Investments and Exponential Functions

Future Value of an Investment with Annual Compounding

If $P$ dollars are invested at an interest rate $r$ per year, compounded annually, the future value $S$ at the end of $t$ years is

$$S = P(1 + r)^t$$

Future Value of an Investment with Periodic Compounding

If $P$ dollars are invested for $t$ years at the annual interest rate $r$, where the interest is compounded $k$ times per year, then the interest rate per period is $\frac{r}{k}$, the number of compounding periods is $kt$, and the future value that results is given by

$$S = P \left(1 + \frac{r}{k}\right)^{kt} \text{ dollars}$$

- Semiannually $\rightarrow k = 2$
- Quarterly $\rightarrow k = 4$
- Monthly $\rightarrow k = 12$
- Daily $\rightarrow k = 360$ or $k = 365$
Future Value of an Investment with Continuous Compounding

If $P$ dollars are invested for $t$ years at an annual interest rate $r$, compounded continuously, then the future value $S$ is given by

$$S = Pe^{rt} \text{ dollars}$$
Daily Versus Annual Compounding of Interest

a. Write the equation that gives the future value of $1000 invested for \( t \) years at 8\% compounded annually.
\[ S = P (1 + r)^t = 1000 \left(1 + 0.08\right)^t = 1000 \left(1.08\right)^t \]

b. Write the equation that gives the future value of $1000 invested for \( t \) years at 8\% compounded daily.
\[ S = P \left(1 + \frac{r}{k}\right)^{kt} = 1000 \left(1 + \frac{0.08}{360}\right)^{360t} \]
\[ \text{assume } k = 360 \]

c. Graph the equations from parts (a) and (b) on the same axes with \( t \) between 0 and 30.

\[ \checkmark \]

d. What is the additional amount of interest earned in 30 years from compounding daily rather than annually?
\[ \text{annually: } S = 1000 \left(1.08\right)^{30} \approx 10,062.66 \]
\[ \text{daily: } S = 1000 \left(1 + \frac{0.08}{360}\right)^{360(30)} \approx 11,020.24 \]
\[ \text{additional interest earned} = 11,020.24 - 10,062.66 = 957.58 \]
Future Value and Continuous Compounding

a. What is the future value of $2650 invested for 8 years at 12\% \text{ compounded continuously}\? \text{ "pert"}

b. How much interest will be earned on this investment?

\[ a) \quad S = Pert = 2650 e^{0.12(8)} \approx 6921 \]

\[ b) \quad \text{interest earned} = \text{future value} - \text{principal} \]
\[ = 6921 - 2650 = 4271 \]
Continuous versus Annual Compounding of Interest

a. For each of 9 years, compare the future value of an investment of $1000 at 8%, compounded annually, and of $1000 at 8%, compounded continuously.

b. Graph the functions for annual compounding and for continuous compounding for $t = 30$ years on the same axes.

c. What conclusion can be made regarding compounding annually and compounding continuously?

\[
a) \quad \text{year } (t) \quad S = 1000 \left(1 + 0.08\right)^t \quad S = 1000e^{0.08t} \\
1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9
\]

Fill out this table using Graph to help you evaluate!
**Doubling Time** Use a spreadsheet, a table, or a graph to estimate how long it takes for an amount to double if it is invested at 6% interest

a. Compounded annually.

\[ S = P (1+r)^t \quad \Rightarrow \quad \frac{2P}{P} = (1.06)^t \]

Doubling time:
\[ t = \frac{\ln 2}{\ln (1+r)} \]
\[ 1.06^t = 2 \]
\[ \ln 1.06^t = \ln 2 \]
\[ t \ln 1.06 = \ln 2 \quad \Rightarrow \quad t = \frac{\ln 2}{\ln 1.06} \approx 11.896 \text{ yrs} \]

b. Compounded continuously.

\[ S = Pe^{rt} \quad \Rightarrow \quad \frac{2P}{P} = e^{0.06t} \]

Doubling time:
\[ t = \frac{\ln 2}{r} \]
\[ e^{0.06t} = 2 \quad \Rightarrow \quad \ln e^{0.06t} = \ln 2 \]
\[ 0.06t \ln e = \ln 2 \quad \Rightarrow \quad t = \frac{\ln 2}{0.06} \approx 11.552 \text{ yrs} \]