Chapter 4: Please review the Algebra Toolbox

Section 4.1: Transformations of Graphs

*Check out the artwork of MC Escher, the "king of transformations"

**Constant Function**
\[ y = f(x) = \text{some number} \]
\[ y = 3 \]

**Identity Function**
\[ f(x) = x \]

```
\begin{align*}
\text{y = 3} & \quad \text{horizontal line} \\
\end{align*}
```

```
\begin{align*}
(2, 2) & \\
(-1, -1) & \\
\end{align*}
```
Basic Quadratic Function
\[ f(x) = x^2 \]

Square Root Function
\[ f(x) = \sqrt{x} = x^{\frac{1}{2}} = \text{sgrt}(x) \]
Basic Cubic Function
\[ f(x) = x^3 \]

Cube Root Function
\[ f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} = \text{root}(3, x) \]
**Absolute Value Function**

\[ f(x) = |x| = \text{abs}(x) \]

- \[ x, \text{ if } x \geq 0 \]
- \[ -x, \text{ if } x < 0 \]

**Greatest Integer Function**

\[ f(x) = [x] = \text{floor}(x) \]

- \([3.7] = 3\)
- \([12] = 12\)
- \([4.999] = 4\)
- \([-3.2] = -4\)

The greatest integer that is less than or equal to \(x\).
Graph Transformations

For a given function $y = f(x)$,

**Vertical Shift**

(y-up/down)

$y = f(x) + k$

Graph is shifted $k$ units up if $k > 0$ and $k$ units down if $k < 0$.

**Horizontal Shift**

(x-left/right)

$y = f(x - h)$

Graph is shifted $h$ units right if $h > 0$ and $h$ units left if $h < 0$.

**Stretch/Compress**

(y-vertical)

$y = af(x)$

Graph is vertically stretched by a factor of $|a|$ if $|a| > 1$.

Graph is compressed by a factor of $|a|$ if $|a| < 1$.

**Reflection**

$y = -f(x)$

Graph is reflected across the x-axis.

$y = f(-x)$

Graph is reflected across the y-axis.
**Ex** Horizontal Shifts

\[ f(x) = \sqrt{x} \]

**Graph**

\[ g(x) = \sqrt{x-2} \quad \text{and} \quad h(x) = \sqrt{x+1} \]

\[ = f(x-2) \]
\[ = f(x-h) \]

where \( h = 2 > 0 \)

Shift \( f \) 2 units right

\[ = f(x+1) \]
\[ = f(x-(-1)) \]
\[ = f(x-h) \]

where \( h = -1 < 0 \)

Shift \( f \) 1 unit left

\( (0,0) \)
\( (2,0) \)
\( (-1,0) \)
**Unconventional Vehicle Sales** The number of E85 flex fuel vehicles, in millions, projected to be sold in the United States can be modeled by the function \( F(x) = 0.084x^{0.675} \), where \( x \) is the number of years after 2000. Convert the function so that \( x \) equals the number of years after 1990.

(Source: www.eia.gov)

<table>
<thead>
<tr>
<th>Years</th>
<th>( x )</th>
<th>( F(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0</td>
<td>0.084(0)^{0.675} = 0 millions</td>
</tr>
<tr>
<td>2001</td>
<td>1</td>
<td>0.084(1)^{0.675} = 0.084 millions</td>
</tr>
<tr>
<td>2005</td>
<td>5</td>
<td>0.084(5)^{0.675} \approx 0.249 millions</td>
</tr>
<tr>
<td>2010</td>
<td>10</td>
<td>( F(10) \approx 0.397 ) millions</td>
</tr>
</tbody>
</table>

\( F(x) = 0.084x^{0.675} \)

\[ G(x) = 0.084(x - 10)^{0.675} \]

\[ G(x) = F(x - 10) \]

\( h = 10 > 0 \) so shift \( F \) 10 units RIGHT.
Exercise 7: Vertical Stretch + Shrink

\[ \text{Graph } g(x) = 2x^2 = 2f(x) \]

\[ \text{Graph } h(x) = \frac{1}{3}x^2 = \frac{1}{3}f(x) \]

\[ h(x) = -3x^2 = -3f(x) \]
Example

Reflections

\[ f(x) = \sqrt{x} \]

Graph \[ g(x) = \sqrt{-x} = f(-x) \]
reflect \( f \) over y-axis

\[ h(x) = -\sqrt{x} = -f(x) \]
reflect \( f \) over x-axis

\[ y = \sqrt{x} \]

Domain: \([0, \infty)\)  
Range: \([0, \infty)\)

\[ y = \sqrt{-x} \]

Domain: \((-\infty, 0]\)  
Range: \([0, \infty)\)

\[ y = -\sqrt{x} \]

Domain: \([0, \infty)\)  
Range: \((-\infty, 0]\)
Ex. Vertical Shifts

\[ f(x) = |x| \]

Graph \[ g(x) = |x| - 2 = f(x) - 2 \]
and \[ h(x) = |x| + 1 = f(x) + 1 \]
Marijuana Use The number of millions of people age 12 and older in the United States who used marijuana during the years 2003 to 2008 is described by the function $M(x) = -0.062(x - 4.8)^2 + 25.4$ for $3 \leq x \leq 8$, where $x$ is the number of years after 2000.

a. The graph of this function is a shifted graph of which basic function? $f(x) = x^2$

b. Find and interpret $M(3)$.

c. Sketch a graph of $y = M(x)$ for $3 \leq x \leq 8$.
(Source: 2008 National Survey on Drug Use and Health, U.S. Department of Health and Human Services)

$M(x) = -0.062(x - 4.8)^2 + 25.4$ quadratic function written in vertex form ...

vertex: $(4.8, 25.4)$ opens ✓ downward b/c lead coeff is negative

Related "Basic" Function: $f(x) = x^2$

b) $M(3)$ represents the number of millions of people in the US age 12 and older who used marijuana in 2003.

$M(3) = -0.062(3 - 4.8)^2 + 25.4 \approx 25.2$ million people

c) Graph Program!
Review Symmetry and Even vs. Odd Functions on your own!

Reflections vs. Symmetry

reflect: verb → you do the manipulation

symmetry: a property of an object; a graph "has symmetry" or "does not have symmetry"

3 types of symmetry for graphs

1. Symmetry about y-axis
2. Symmetry about x-axis
3. Symmetry about the origin
Symmetry about y-axis

- The left and right "sides" (relative to the y-axis) are "the same."

- If \((a, b)\) is on the graph, then so is \((-a, b)\).
Symmetry about $x$-axis

* The top and bottom (relative to the $x$-axis) are the same.

* If $(a, b)$ is on the graph, then so is $(a, -b)$.

* Note that any graph that is symmetric about the $x$-axis is NOT a function (except for a graph where every point on the graph has $y$-coord 0).
Symmetry about the Origin

If you ROTATE the graph 180°, the “new graph” looks just like the original graph.

If \((a, b)\) is on the graph, then so is \((-a, -b)\).

Also symmetric about the x-axis and the y-axis.
**Even vs. Odd Functions**

* A function is **even** if \( f(-x) = f(x) \); if \((a,b)\) is on the graph of \(f\), then so is \((-a,b)\).
  
  * If \(f\) is even, the graph of \(f\) will be symmetric about the \(y\)-axis.

* A function is **odd** if \( f(-x) = -f(x) \); if \((a,b)\) is on the graph of \(f\), then so is \((-a,-b)\).
  
  * If \(f\) is odd, the graph of \(f\) will be symmetric about the origin.
Ex Even, odd, or neither?

a) \( f(x) = 3x^4 - 5x^2 + 6 \)

\[
f(-x) = 3(-x)^4 - 5(-x)^2 + 6 = 3x^4 - 5x^2 + 6 = f(x)
\]

\( \star \) Since \( f(-x) = f(x) \), this function is even.

b) \( f(x) = 5x^3 - 7x^2 + 2x - 3 \)

\[
f(-x) = 5(-x)^3 - 7(-x)^2 + 2(-x) - 3 = -5x^3 - 7x^2 - 2x - 3 \\
\neq f(x) \quad \text{and} \quad \neq -f(x)
\]

\( \star \) Since \( f(-x) \neq f(x) \) and \( f(-x) \neq -f(x) \), this function is neither even nor odd.
c) \( f(x) = 4x^7 - 6x^3 + 12x \)

\[
f(-x) = 4(-x)^7 - 6(-x)^3 + 12(-x) \\
= -4x^7 + 6x^3 - 12x \\
= -f(x)
\]

\* Since \( f(-x) = -f(x) \), this function is odd.