Section 3.3: Piecewise-Defined Functions and Power Functions

A piecewise-defined function: a function that is "broken" into "pieces".

\[
\text{Example: } f(x) = \begin{cases} 
3x + 1, & \text{if } x < 0 \\
x^2, & \text{if } x \geq 0
\end{cases}
\]

\[f(-1) = 3(-1) + 1 = -2\]
\[f(-2) = 3(-2) + 1 = -5\]
\[f(0) = 0^2 = 0\]
\[f(1) = 1^2 = 1\]
\[f(2) = 2^2 = 4\]
Postal Rates

<table>
<thead>
<tr>
<th>Weight Not Over (ounces)</th>
<th>Cost is ( C(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0.49</td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>0.93</td>
</tr>
<tr>
<td>3.5^1</td>
<td>1.15</td>
</tr>
</tbody>
</table>

\[
C(x) = \begin{cases} 
  \$0.49, & \text{if } 0 < x \leq 1 \\
  \$0.71, & \text{if } 1 < x \leq 2 \\
  \$0.93, & \text{if } 2 < x \leq 3 \\
  \$1.15, & \text{if } 3 < x \leq 3.5 
\end{cases}
\]

* Graph the function

* This type of piecewise-defined function is called a step function.
Exercise

Graph: \[ f(x) = \begin{cases} 5x + 2, & 0 \leq x < 3 \\ x^3, & 3 \leq x \leq 5 \end{cases} \]

Compute:

1. \( f(-1) \) is undefined
2. \( f(0) = 5(0) + 2 = 2 \)
3. \( f(1) = 5(1) + 2 = 7 \)
4. \( f(2) = 5(2) + 2 = 12 \)
5. \( f(3) = 3^3 = 27 \)
6. \( f(4) = 4^3 = 64 \)
7. \( f(5) = 5^3 = 125 \)
8. \( f(6) \) is undefined
**Absolute Value Function**

\[ f(x) = |x| = \text{abs}(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases} \]

| x  | |x| |
|----|----|
| -2 | 2  |
| -1 | 1  |
| 0  | 0  |
| 1  | 1  |
| 2  | 2  |
| 3  | 3  |
Ex: Graph \( f(x) = |x-4| \)

| \( x \)  | \( |x-4| \)  |
|------|-------|
| -3   | 7     |
| -2   | 6     |
| -1   | 5     |
| 0    | 4     |
| 1    | 3     |
| 2    | 2     |
| 3    | 1     |
| 4    | 0     |
| 5    | 1     |
| 6    | 2     |
| 7    | 3     |

b) Compute \( f(-2) = 6 \) and \( f(5) = 1 \)

c) What is the domain of \( f(x) = |x-4| \)? All real numbers: \( \mathbb{R} = (-\infty, \infty) \)

What is the range? \( [0, \infty) \)
Exercise Find the domain and range of \( f(x) = \frac{1}{x+2} \).

- **Domain:** all reals except \(-2\): \( \exists x : x \neq -2 \),
or \( (-\infty, -2) \cup (-2, \infty) \)

- **Range:** since \( \frac{1}{x+2} \neq 0 \), range is \( \exists y : y > 0 \),
= \( (0, \infty) \)

- \( f \) is increasing on \( (-\infty, -2) \)
- \( f \) is decreasing on \( (-2, \infty) \)
Power Functions

**power function**: \( f(x) = a \cdot x^b \), where \( a, b \) are real numbers and \( b \neq 0 \).

- \( y = mx \) — linear function
- \( y = ax^2 \) — quadratic function
- \( y = ax^3 \) — cubic power function
Squaring Function \[ y = x^2 \]
Cubing Function  \[ y = x^3 \]
Square Root Function \[ y = \sqrt{x} = x^{\frac{1}{2}} \quad \text{domain: } [0, \infty) \]
Cube Root Function

\[ y = \sqrt[3]{x} = x^{\frac{1}{3}} \]

domain: \((-\infty, \infty)\)
Reciprocal Function

Graph is called a hyperbola.

\[ y = \frac{1}{x} = x^{-1} \]

Domain: \((-\infty, 0) \cup (0, \infty)\)

Range: \((-\infty, 0) \cup (0, \infty)\)

Horizontal asymptote at the line \(y = 0\)

Vertical asymptote at the line \(x = 0\)
**Harvesting** A farmer’s main cash crop is tomatoes, and the tomato harvest begins in the month of May. The number of bushels of tomatoes harvested on the $x$th day of May is given by the equation $B(x) = 6(x + 1)^{3/2}$. How many bushels did the farmer harvest on May 8?

$B(x) = 6(x + 1)^{3/2}$

Compute $B(8) = 6(8 + 1)^{3/2}$

$= 6 \cdot 9^{3/2}$

$= 6 \cdot (\sqrt[3]{9})^3$

$= 6 \cdot 3^3 = 6 \cdot 27 = 162$
**Taxi Miles**  The Inner City Taxi Company estimated, on the basis of collected data, that the number of taxi miles driven each day can be modeled by the function $Q = 489L^{0.6}$, when they employ $L$ drivers per day.

**a.** Graph this function for $0 \leq L \leq 35$.

**b.** How many taxi miles are driven each day if there are 32 drivers employed?

**c.** Does this model indicate that the number of taxi miles increases or decreases as the number of drivers increases? Is this reasonable?

\[ Q(L) = 489L^{0.6} \]

is a power function

\[ f(x) = 489x^{0.6} \text{ over } [0, 35] \]

Done in Graph!

b) \[ Q(32) = f(32) = 489 \cdot 32^{0.6} = 3912 \text{ miles driven by 32 drivers} \]

c) It appears that as the number of drivers increases, so does the number of taxi miles driven.