Section 2.3: Systems of Linear Equations

Solve \[ \begin{align*}
2x + 3y &= -1 \quad (1) \\
x - 4y &= -6 \quad (2)
\end{align*} \]

Find the point of intersection of the two graphs. From graph, it looks like the point of intersection is \((-2, 1)\).

Substitution

- Solve one eqn for one of the variable: \[ x = 4y - 6 \]

- Substitute this into the other eqn: \[ 2(4y - 6) + 3y = -1 \]

- Solve the resulting equation: \[ 8y - 12 + 3y = -1 \]

\[ \begin{align*}
11y - 12 &= -1 \\
11y &= 11 \\
y &= 1
\end{align*} \]

Solution: \( x = -2, y = 1 \) Solution set: \( \{(-2, 1)\} \)

- Substitute this and solve for other variable: \[ x = 4(1) - 6 = -2 \]
2. Elimination: make a variable "go away"

eliminate x: \[\begin{align*}
2x + 3y &= -1 \\
x - 4y &= -6
\end{align*}\] \[\xrightarrow{\times (-2)}\] \[\begin{align*}
2x + 3y &= -1 \\
-2x + 8y &= 12
\end{align*}\]

\[\begin{align*}
y &= 1 \\
y &= 1 \quad \text{now, find } x!
\end{align*}\]

eliminate y: \[\begin{align*}
2x + 3y &= -1 \quad \xrightarrow{\times 4} \quad 8x + 12y = -4 \\
x - 4y &= -6 \quad \xrightarrow{\times 3} \quad 3x - 12y = 18
\end{align*}\]

\[\begin{align*}
x &= -22 \\
x &= -2 \quad \text{now, find } y!
\end{align*}\]
(Ex) Solve $\begin{align*}
2x - 4y &= 6 \\
3x + 5y &= 20
\end{align*}$

\[ \times 5 \rightarrow \begin{align*}
10x - 20y &= 30 \\
12x + 20y &= 80
\end{align*} \]

Solution set: $\{(5, 1)\}$

Consistent system: the system of equations has at least one solution. For linear functions: two choices:

\[ \times \]

1. exactly one solution
2. infinitely many solutions

\[ \times \]

inconsistent system: the system of equations has NO SOLUTION

L the two lines are parallel!
\[ \begin{align*}
\text{Ex. } & \quad \text{Solve} \quad 2x - y = 7 \quad \longrightarrow \quad 4x - 2y = 14 \\
& \quad -4x + 2y = -14 \quad \longrightarrow \quad -8x + 4y = -28 \\
\end{align*} \]

these are really the same line!

there are infinitely many solutions;
all \( y \) the solutions satisfy \( y = 2x - 7 \) all real numbers

Solution set: \( \{ (x, 2x-7) : x \in \mathbb{R} \} \) is an element of
A manufacturer of reading lamps has total revenue given by $R = 15.80x$ and total cost given by $C = 8593.20 + 3.20x$, where $x$ is the number of units produced and sold. Use a non-graphical method to find the number of units that gives break-even for this product.

Break-Even Analysis: try to find value of $x$ where revenue = cost.

\[
R = C
\]
\[
15.80x = 8593.20 + 3.20x
\]
\[
12.60x = 8593.20
\]
\[
x = 682 \text{ units}
\]
Section 2.3 (cont.)

**Market Equilibrium**

Suppose that the daily demand for a product is given by \( p = 200 - 2q \), where \( q \) is the number of units demanded and \( p \) is the price per unit in dollars, and that the daily supply is given by \( p = 60 + 5q \), where \( q \) is the number of units supplied and \( p \) is the price in dollars. If a price results in more units being supplied than demanded, we say there is a surplus, and if the price results in fewer units being supplied than demanded, we say there is a shortfall. **Market equilibrium** occurs when the supply quantity equals the demand quantity (and when the prices are equal)—that is, when \( q \) and \( p \) both satisfy the system

\[
\begin{align*}
\text{demand eqn} \quad \{ & p = 200 - 2q \\
\text{supply eqn} \quad & p = 60 + 5q 
\end{align*}
\]

**a.** If the price is $140, how many units are supplied and how many are demanded?

**b.** Does this price give a surplus or a shortfall of the product?

**c.** What price gives market equilibrium?

**c) Solve**

\[
\begin{align*}
p &= 200 - 2q \\
p &= 60 + 5q
\end{align*}
\]

\[
\begin{align*}
solve \quad 200 - 2q &= 60 + 5q \\
140 &= 7q \\
q &= 20 \text{ units supplied/demanded}
\end{align*}
\]

But, \( p = 200 - 2(20) = 1\)60

\[
\begin{align*}
solve \quad 140 &= 60 + 5q \\
80 &= 5q \\
q &= 16
\end{align*}
\]

At $140, the demand will be 30 units.

At $140, the supply will be 16 units.
(Ex) Solve \[3x - 2y = 5 \quad \text{and} \quad 9x - 6y = 15\]

\[-9x + 6y = 32 \quad \rightarrow \quad -9x + 6y = 32\]

There is **No Solution**.

The system is **inconsistent**.

These lines are **parallel**.

0 = 47 this is **NEVER TRUE**!