Significant Figures

There are two types of numbers that are encountered in science. One type of number is an exact number. This is like a count of some items. If I have 3 pencils sitting on my desk, the number of pencils is exact, there is no uncertainty in that number. This type of number also occurs often as a part of a definition. There are 12 inches in a foot. There are exactly 12 inches in a foot, it can never be 13 or 12 ½ or 12.3 inches. Exact numbers do not cause us problems.

The second type of number is a measured number or measurement. For example, if I measure my height, I will use some type of measuring tool to accomplish the task. Whenever I use a measuring tool, the number I get as an answer always has some uncertainty associated with it. If my ruler has markings every inch, I can probably measure my height, certain to the nearest inch, but if I fall in between two markings I can approximate the fraction of an inch the measurement includes. Perhaps I am 71.5 inches tall. In general, we can read a measuring instrument to the nearest tenth of the smallest marked increment on the device. The fractional part here is somewhat of a guess and therefore it is not a number known with certainty. We explain this difference in a concept known as uncertainty of a measurement. This is true for all measurements.

If I use a measuring device that has more increments, I may be able to improve the value for my height, but I will always come to a point where I am estimating the last figure. Once again, we can read a measuring instrument to the nearest tenth of the smallest marked increment on the device. It is important for the scientist to keep track of this uncertainty in the measurements made as they have an effect on the limit to which we can report a result. Our method for keeping track of this uncertainty and utilizing it correctly to manipulate numbers is called significant figures.

Most students do not understand significant figures the first time they are introduced. Rarely do students enjoy significant figures, but they are a necessary part of dealing with measurements and calculations.

Identifying Significant Figures in a Number
The first aspect to understanding this process is to simply identify the significant figures in a given number. Again, most students find this difficult, at least initially, but it is really nothing more than learning a few rules about digits in a number. Here are the rules:

1. All non-zero digits are significant.
2. Zeros to the left of a number (these are called leading zeros) are not significant.

Example: These leading zeros will appear in decimal numbers of less than one.
So, in 0.0076 the leading zeros are not significant. In 0.00011, none of the zeros are significant.

3. **Zeros between non-zero digits** are significant.
   Example: In the numbers 2009, 404, 212109, the zeros are all significant.

The only remaining rule is when the zeros fall at the end of a number. This one is a little tricky and we break rule #4 into two parts for easier comprehension.

4. **Zeros at the end (right) of a number…**
   A. If the decimal point is visible, then the zero(s) **are** significant.
   
   Example: 90.00, 4.50, 1022.600. In all cases, all the zeros are significant.
   
   B. If the decimal point is not visible, then the zero(s) are **not** significant.
   
   Example: i) 90,000 and 1020  The zeros at the end of the number are not significant.

Once the significant figures have been determined we can conclude that:

*The uncertainty resides in the last significant figure.*

**Using Significant Figures in Calculations**

Why do we need to identify significant figures? Have you ever worked a math problem where the author told you to “round off your answer to the nearest tenth?” How does one know how to round an answer? Significant figures allow us to round answers to the proper place to reflect the correct uncertainty of our measurements.

To understand the use of significant figures as they apply to calculations, it is best to separate the rules into two sets of operations: addition & subtraction and multiplication & division.

**Rules for Addition & Subtraction**

This rule states that the answer to an addition or subtraction problem will have its uncertainty in the same place as the number that has the largest uncertainty involved in the calculation. Phew, what does this mean? Consider the calculation shown below.

\[
\begin{array}{c}
2030 \\
+ \ 14 \\
\hline
2040
\end{array}
\]

The first number, 2030, has three significant figures (the final zero is not significant). This means the uncertainty is in the last significant figure, which is the 3 or the “tens” place. In the second number, 14, there are two significant figures so the uncertainty is in the 4 or the “units” place. Once the numbers are added, the Addition & Subtraction rule says that the final uncertainty must be in the “tens” places because that uncertainty is larger than the “units” place.
Therefore, we must round our number off to the nearest “tens” or a final answer of 2040 where the final zero is not significant.

**Rules for Multiplication and Division**

This rule states that the answer to a multiplication or division problem the answer will have the same number of significant figures as the least number of significant figures involved in the calculation. This can be confusing, but it gets easier with practice. Consider the calculation shown below.

\[
14.3 \\
\times 9.1 \\
130
\]

If the calculation is done on a calculator, the result would be 130.13. However, the number 14.3 has three significant figures and the number 9.1 has two significant figures. This limits our answer to only two significant figures. The final result listed is 130, the value rounded to only two significant figures.